Math 115
Winter 2017
Lecture 16

① Simplify: $\dfrac{x^2-4}{x^2+5x+6} = \dfrac{(x+2)(x-2)}{(x+2)(x+3)} = \boxed{\dfrac{x-2}{x+3}}$

② Find all excluded values: $\dfrac{x^2+2x+1}{3x^2-x-14}$

Denominator $= 0$

$3x^2 - x - 14 = 0$

$ax^2 + bx + c = 0$

$a=3,\quad b=-1,\quad c=-14$

$b^2-4ac = (-1)^2 - 4(3)(-14)$

$= 169$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{-(-1) \pm \sqrt{169}}{2(3)} = \dfrac{1 \pm 13}{6}$

$x = \dfrac{1+13}{6} = \dfrac{14}{6} = \dfrac{7}{3}$

$x = \dfrac{1-13}{6} = \dfrac{-12}{6} = -2$

E.V. $= \dfrac{7}{3}, -2$

① Reduce: $\dfrac{2x^2-6x+4}{4x^2+12x-16} = \dfrac{2(x^2-3x+2)}{4(x^2+3x-4)} = \dfrac{2(x-1)(x-2)}{4(x+4)(x-1)}$

$= \boxed{\dfrac{x-2}{2(x+4)}}$

② Find all excluded values: $\dfrac{x^2+x-20}{2x^2+11x-6}$

Excluded values: Deno. $=0$

$2x^2+11x-6=0$

$a=2 \quad b=11 \quad c=-6$

$b^2-4ac=(11)^2-4(2)(-6)$

$=121+48=169$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{-11\pm\sqrt{169}}{2(2)}=\dfrac{-11\pm13}{4}$

$x=\dfrac{-11+13}{4}=\dfrac{2}{4}=\dfrac{1}{2}$

$x=\dfrac{-11-13}{4}=\dfrac{-24}{4}=-6$

E.V. $\dfrac{1}{2}, -6$

Simplify:

① $\dfrac{10x+20}{2x^2-3x+1}\cdot\dfrac{x^2-1}{5x+10}$

$=\dfrac{2\cancel{10(x+2)}}{(2x-1)\cancel{(x-1)}}\cdot\dfrac{(x+1)\cancel{(x-1)}}{\cancel{5(x+2)}}$

$=\boxed{\dfrac{2(x+1)}{2x-1}}$

② $\dfrac{x^2-2x-3}{x^2-4}\div\dfrac{x+1}{2x+4}$

$=\dfrac{x^2-2x-3}{x^2-4}\cdot\dfrac{2x+4}{x+1}$

$=\dfrac{(x-3)\cancel{(x+1)}}{\cancel{(x+2)}(x-2)}\cdot\dfrac{2\cancel{(x+2)}}{\cancel{x+1}}$

$=\boxed{\dfrac{2(x-3)}{x-2}}$

Simplify:

$$\frac{x^2-14x+49}{2x^2-3x-14} \div \frac{3x^2-20x-7}{x^2-6x-16}$$

$$= \frac{(x-7)(x-7)}{(2x-7)(x+2)} \cdot \frac{(x-8)(x+2)}{(3x+1)(x-7)}$$

$$= \frac{(x-7)(x-8)}{(2x-7)(3x+1)}$$

Simplify:

$$\frac{x^2-7x}{x^2+8x+16} + \frac{6x-20}{x^2+8x+16} = \frac{x^2-7x+6x-20}{x^2+8x+16}$$

$$= \frac{x^2-x-20}{x^2+8x+16} = \frac{(x-5)(x+4)}{(x+4)(x+4)} = \boxed{\frac{x-5}{x+4}}$$

$$\frac{5-3x}{x^2-2x+1} - \frac{x+1}{x^2-2x+1} = \frac{5-3x-x-1}{x^2-2x+1}$$

$$= \frac{-4x+4}{x^2-2x+1} = \frac{-4(x-1)}{(x-1)(x-1)} = \boxed{\frac{-4}{x-1}}$$

Simplify:

$$\frac{x-9}{x^2+3x-4} - \frac{2x-5}{x^2+3x-4} = \frac{x-9-2x+5}{x^2+3x-4} = \frac{-x-4}{x^2+3x-4} = \frac{-1(x+4)}{(x+4)(x-1)} = \frac{-1}{x-1}$$

$$\frac{x-6}{x^2+5x+6} + \frac{9}{x^2+5x+6} = \frac{x-6+9}{x^2+5x+6} = \frac{x+3}{x^2+5x+6} = \frac{1\ x+3}{(x+3)(x+2)} = \frac{1}{x+2}$$

The product of two cons. odd integers is 35.
Find all such integers.

$\rightarrow x$ & $x+2$

Product = 35

$x(x+2) = 35$

$x^2 + 2x = 35$

$x^2 + 2x - 35 = 0$

→ Let's use Q-formula to Solve

$a=1,\ b=2,\ c=-35$

$b^2-4ac = 2^2-4(1)(-35) = 144$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{144}}{2} = \frac{-2 \pm 12}{2}$$

$x = \frac{-14}{2} = -7$, $x = \frac{10}{2} = 5$

x	$x+2$
5	7
-7	-5

The product of two cons. even integers is 48.
Find all such integers.

x & $x+2$

6 & 8
-8 & -6

$x(x+2)=48$

$x^2+2x-48=0$

$a=1 \quad b=2 \quad c=-48$

$b^2-4ac=(2)^2-4(1)(-48)$

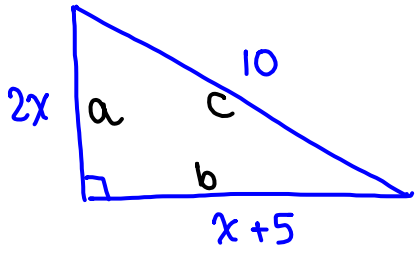
$=196$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-2\pm\sqrt{196}}{2}=\frac{-2\pm14}{2}$

$x=\frac{-2+14}{2}=6$

$x=\frac{-2-14}{2}=-8$

Find x:



Right Triangle

Pythagorean Thrm

$a^2+b^2=c^2$

$(2x)^2+(x+5)^2=10^2$

$4x^2+x^2+10x+25=100$

$5x^2+10x+25-100=0$

$5x^2+10x-75=0$

Divide by 5

$x^2+2x-15=0$

$a=1, b=2, c=-15$

$b^2-4ac=(2)^2-4(1)(-15)$

$=4+60=64$

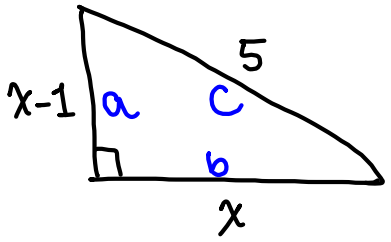
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-2\pm\sqrt{64}}{2}=\frac{-2\pm8}{2}$

~~$x=-5$~~

$x=3$

Find x.



Right Triangle

Pythagorean Thm

$a^2 + b^2 = c^2$

$(x-1)^2 + x^2 = 5^2$

$x^2 - 2x + 1 + x^2 = 25$

$2x^2 - 2x + 1 - 25 = 0$

$2x^2 - 2x - 24 = 0$

Divide by 2

$x^2 - x - 12 = 0$

$a=1,\ b=-1,\ c=-12$

$b^2 - 4ac = (-1)^2 - 4(1)(-12) = 49$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{-(-1) \pm \sqrt{49}}{2(1)} = \dfrac{1 \pm 7}{2}$

$x = 4$ ~~$x = -3$~~ $\rightarrow \{4\}$

Adding & Subtracting fractions with different denominators:

$$\frac{5}{x-2} + \frac{3}{x+4} = \frac{5(x+4)}{(x-2)(x+4)} + \frac{3(x-2)}{(x+4)(x-2)}$$

$$= \frac{5(x+4)}{(x-2)(x+4)} + \frac{3(x-2)}{(x+4)(x-2)} = \frac{5x+20+3x-6}{(x+4)(x-2)}$$

$$= \frac{8x+14}{(x+4)(x-2)} = \frac{2(4x+7)}{(x+4)(x-2)}$$

Simplify:

$$\frac{4}{x-5} - \frac{2}{x+3} = \frac{4(x+3)}{(x-5)(x+3)} - \frac{2(x-5)}{(x+3)(x-5)}$$

$$= \frac{4x+12-2x+10}{(x-5)(x+3)} = \boxed{\frac{2x+22}{(x-5)(x+3)}} = \boxed{\frac{2(x+11)}{(x-5)(x+3)}}$$

$$\frac{3}{(x-2)(x+3)} - \frac{2}{(x+3)(x-4)} = \boxed{\frac{x-8}{(x-2)(x+3)(x-4)}}$$

$$\frac{3(x-4)}{(x-2)(x+3)(x-4)} - \frac{2(x-2)}{(x-2)(x+3)(x-4)} = \frac{3x-12-2x+4}{(x-2)(x+3)(x-4)}$$

Simplify

$$\frac{5}{x^2+5x+6} - \frac{4}{x^2-9}$$

$$= \frac{5(x-3)}{(x+2)(x+3)(x-3)} - \frac{4(x+2)}{(x-3)(x+3)(x+2)}$$

$$= \frac{5x-15-4x-8}{(x+2)(x+3)(x-3)} = \boxed{\frac{x-23}{(x+2)(x+3)(x-3)}}$$

Simplify:

$$\frac{x}{x^2+9x+20} - \frac{4}{x^2+7x+12} = \frac{x}{(x+5)(x+4)} - \frac{4}{(x+3)(x+4)}$$

$$= \frac{x(x+3)}{(x+5)(x+4)(x+3)} - \frac{4(x+5)}{(x+3)(x+4)(x+5)}$$

$$= \frac{x^2+3x-4x-20}{(x+5)(x+4)(x+3)} = \frac{(x-5)(x+4)}{(x+5)(x+4)(x+3)} = \frac{x^2-x-20}{(x+5)(x+4)(x+3)}$$

$$= \boxed{\frac{x-5}{(x+5)(x+3)}}$$

Simplify

$$\frac{x}{x^2+5x+6} - \frac{2}{x^2+3x+2} =$$

$$\frac{x(x+1)}{(x+2)(x+3)(x+1)} - \frac{2(x+3)}{(x+2)(x+1)(x+3)} =$$

$$\frac{x^2+x-2x-6}{(x+2)(x+3)(x+1)} = \frac{x^2-x-6}{(x+2)(x+3)(x+1)}$$

$$= \frac{(x-3)(x+2)}{(x+2)(x+3)(x+1)} = \boxed{\frac{x-3}{(x+3)(x+1)}}$$

Simplify:

$$\frac{3x}{x^2-4x+4}+\frac{10}{x^2+x-6}$$

$P=-60$, $S=19$

$-1, 60$
$-2, 30$
$-3, 20$
$-4, 15$
$-5, 12$
$-6, 10$

$$=\frac{3x(x+3)}{(x-2)(x-2)(x+3)}+\frac{10(x-2)}{(x-2)(x+3)(x-2)}$$

$$=\frac{3x^2+9x+10x-20}{(x-2)(x-2)(x+3)}=\boxed{\frac{3x^2+19x-20}{(x-2)(x-2)(x+3)}}$$

Solve: $\frac{5}{8}-\frac{3}{5}=\frac{x}{10}$ LCD = 40

$$\cancel{40}^{5}\cdot\frac{5}{\cancel{8}}-\cancel{40}^{8}\cdot\frac{3}{\cancel{5}}=\cancel{40}^{4}\cdot\frac{x}{\cancel{10}}$$

$$25-24=4x$$

$$1=4x \rightarrow \boxed{x=\frac{1}{4}} \Rightarrow \left\{\frac{1}{4}\right\}$$

Solve $x+\frac{3}{x}=-4$

$x\cdot x+x\cdot\frac{3}{x}=x\cdot(-4)$

LCD = x

$x^2+3=-4x$

$x^2+4x+3=0$

$(x+3)(x+1)=0$

Z.P.R.

$x=-3, x=-1$

$\{-3,-1\}$

Solve

$x + \frac{8}{x} = -9$

LCD = x

$x^2 + 8 = -9x$

$x^2 + 9x + 8 = 0$

$(x+8)(x+1) = 0$

$x = -8$ $x = -1$

$\{-8, -1\}$

$2 + \frac{5}{x} = \frac{12}{x^2}$

LCD = x^2

$2x^2 + 5x = 12$

$2x^2 + 5x - 12 = 0$

$(2x-3)(x+4) = 0$

by Z.P.R.

$x = 3/2$ $x = -4$

$\{-4, 3/2\}$

The sum of a number and its reciprocal is $\frac{5}{2}$.

Find such number.

Let x be the number

So reciprocal is $\frac{1}{x}$

$x + \frac{1}{x} = \frac{5}{2}$

LCD = $2x$

$2x \cdot x + 2x \cdot \frac{1}{x} = 2x \cdot \frac{5}{2}$

$2x^2 + 2 = 5x$

$2x^2 - 5x + 2 = 0$

$(2x-1)(x-2) = 0$

by Z.P.R.

$x = \frac{1}{2}$ or $x = 2$

$\frac{1}{2}$ or 2

Solve

$$\frac{x}{x+5} - \frac{5}{x-5} = \frac{14}{x^2-25}$$

$LCD = (x+5)(x-5)$

$$(x+5)(x-5)\cdot\frac{x}{x+5} - (x+5)(x-5)\cdot\frac{5}{x-5} = (x+5)(x-5)\cdot\frac{14}{x^2-25}$$

$$x(x-5) - 5(x+5) = 14$$

$$x^2 - 5x - 5x - 25 - 14 = 0$$

$$x^2 - 10x - 39 = 0$$

$$(x+3)(x-13) = 0$$

$\{-3, 13\}$

Solve

$$\frac{3}{x+4} - \frac{2}{x-4} = \frac{-16}{x^2-16}$$

Excluded Value

$LCD = (x+4)(x-4)$

$x = 4$

$$3(x-4) - 2(x+4) = -16$$

$$3x - 12 - 2x - 8 = -16$$

$$x - 20 = -16$$

$$x = -16 + 20$$

$\emptyset$

No Soln.

Solve by using Cross-Multiplication:

$$\frac{3}{x-4} = \frac{5}{x+1}$$

$5(x-4) = 3(x+1)$

$5x - 20 = 3x + 3$

$5x - 3x = 3 + 20$

$2x = 23$

$x = 23/2$

$\{23/2\}$

$$\frac{3}{x+7} = \frac{x+10}{x+7}$$

$(x+10)(x+7) = 3(x+7)$

$x^2 + 7x + 10x + 70 = 3x + 21$

$x^2 + 17x + 70 - 3x - 21 = 0$

$x^2 + 14x + 49 = 0$

$(x+7)(x+7) = 0$

by Z.P.R. $x = -7$ is an E.V.

$\emptyset$

Solve by using Cross-Multiplication

1) $$\frac{y+3}{y-3} = \frac{6}{y-3}$$

$(y+3)(y-3) = 6(y-3)$

$y^2 - 9 = 6y - 18$

$y^2 - 9 - 6y + 18 = 0$

$y^2 - 6y + 9 = 0$

$(y-3)(y-3) = 0$

~~$y = 3$~~ is an E.V.

$\emptyset$

2) $$\frac{x+1}{x+2} = \frac{x-3}{x+1}$$

$(x+1)(x+1) = (x+2)(x-3)$

~~$x^2 + x + x + 1 = x^2 - 3x + 2x - 6$~~

$2x + 1 = -x - 6$

$2x + x = -6 - 1$

$3x = -7$

$\boxed{x = \frac{-7}{3}}$

$\left\{\frac{-7}{3}\right\}$

Solve: $\frac{5}{x+1} + \frac{2x}{x^2-1} = \frac{1}{x+1}$

LCD $= (x+1)(x-1)$ E.N.: $-1, 1$

$5(x-1) + 2x = 1(x-1)$

$5x - 5 + 2x = x - 1$

$7x - 5 = x - 1$

$7x - x = -1 + 5$

$6x = 4$

$x = \frac{4}{6}$

$x = \frac{2}{3}$

Solve:

$$\frac{3}{x^2+4x+3} - \frac{2}{x^2+3x+2} = \frac{1}{x^2+5x+6}$$

$$\frac{3}{(x+3)(x+1)} - \frac{2}{(x+2)(x+1)} = \frac{1}{(x+2)(x+3)}$$

LCD $= (x+3)(x+1)(x+2)$ E.N.: $-1, -2, -3$

$3(x+2) - 2(x+3) = 1(x+1)$

$3x + 6 - 2x - 6 = x + 1$

$x = x + 1$

$x - x = 1$

$0 = 1$

$\emptyset$

$$\frac{x}{x+2} + \frac{2x}{x-2} = \frac{x^2}{x^2-4}$$

LCD $= (x+2)(x-2)$, E.V. $-2, 2$

$$x(x-2) + 2x(x+2) = x^2$$

$$x^2 - 2x + 2x^2 + 4x = x^2$$

$$2x^2 + 2x = 0 \rightarrow x^2 + x = 0$$

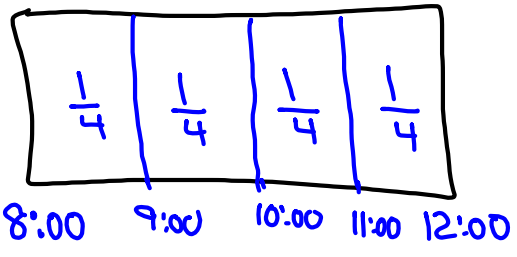
Divide by 2

$$x(x+1) = 0$$

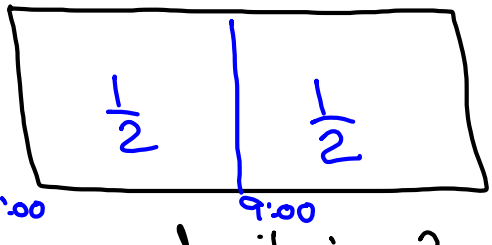
$x = 0$, $x = -1$

$\{-1, 0\}$

Word Problem:



Jose can paint this wall in 4 hrs.



I can do it in 2 hrs.

How long if we work together?

To do a Complete work,

Work by Jose	+	Work by me	=	one complete work

$\frac{1}{4} \cdot t + \frac{1}{2} \cdot t = 1 \Rightarrow \frac{t}{4} + \frac{t}{2} = 1$

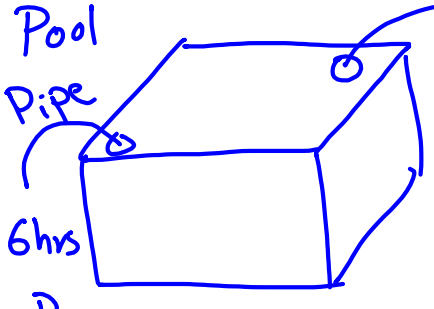
LCD = 4

$t + 2t = 4$

$3t = 4$

$t = \frac{4}{3}$

1 hr 20 Mins



Pipe can fill it up in 8 hrs. A

How long if they are both working, Pool was empty, drain is closed?

Work by Pipe A	+	Work by Pipe B	=	1

$\frac{1}{8} \cdot t + \frac{1}{6} \cdot t = 1$

$\frac{t}{8} + \frac{t}{6} = 1$

LCD = 24

$3t + 4t = 24$

$7t = 24$

$t = \frac{24}{7}$

$t \approx 3.4$

3.4 hrs

Sink:

Work by cold water + Work by drain = 1 comp. work

$\frac{1}{8} \cdot t - \frac{1}{10} \cdot t = 1$

LCD = 40 → $5t - 4t = 40$

$t = 40$

40 mins.

Closed drain, cold water can fill it up in 8 mins.

Drain can empty the sink in 10 mins.

Water is running, drain is open, how long to fill it up?

Jose and I can paint a wall in 2 hrs if we work together.

Jose works twice as fast as me.

How long for each one of us to complete the work if we work alone.

Jose	Me
1 hr	2 hrs
4 hrs	8 hrs
10 hrs	20 hrs
x hrs	2x hrs

Work by Jose + Work by me = 1 complete work

$\frac{1}{x} \cdot 2 + \frac{1}{2x} \cdot 2 = 1$

$\frac{2}{x} + \frac{2}{2x} = 1$ → $\frac{2}{x} + \frac{1}{x} = 1$

LCD = x

$x \cdot \frac{2}{x} + x \cdot \frac{1}{x} = x \cdot 1$

$2 + 1 = x$

$\boxed{x = 3}$ Jose → 3 hrs

I can → 6 hrs.

Find two cons. odd integers such that the sum of their reciprocals is $\frac{8}{15}$.

Two cons. odd integers x & $x+2$

What are their reciprocals? $\frac{1}{x}$ & $\frac{1}{x+2}$

Sum of reciprocals is $\frac{8}{15}$

$\boxed{\frac{1}{x}} + \boxed{\frac{1}{x+2}} = \boxed{\frac{8}{15}}$ LCD $= x \cdot (x+2) \cdot 15$

$15(x+2) + 15x = 8x(x+2)$

$15x + 30 + 15x = 8x^2 + 16x$

$30x + 30 = 8x^2 + 16x$

$8x^2 + 16x - 30x - 30 = 0$

$8x^2 - 14x - 30 = 0$

Divide by 2.

$4x^2 - 7x - 15 = 0$

$4x^2 - 7x - 15 = 0$

$a = 4 \quad b = -7 \quad c = -15$

$b^2 - 4ac = (-7)^2 - 4(4)(-15) = 289$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-7) \pm \sqrt{289}}{2(4)}$$

$$= \frac{7 \pm 17}{8}$$

$x = \frac{7+17}{8} = 3$

$x = \frac{7-17}{8} = \frac{-5}{4}$ is not an integer.

$3 \& 5$

$\frac{1}{3} + \frac{1}{5} = \frac{5+3}{15} = \frac{8}{15}$ ✓